Diamagnetic levitation: Flying frogs and floating magnets (invited)

M. D. Simon[a]
Department of Physics and Astronomy, University of California, Los Angeles, Los Angeles, California 90095

A. K. Geim[b]
High Field Magnet Laboratory, University of Nijmegen, The Netherlands

Contrary to our intuition, apparently nonmagnetic substances can be levitated in a magnetic field and can stabilize free levitation of a permanent magnet. Most substances are weakly diamagnetic and the tiny forces associated with this property make the two types of levitation possible. Living things mostly consist of diamagnetic molecules (such as water and proteins) and components (such as bones) and therefore can be levitated and can experience low gravity. In this way, frogs have been able to fly in the throat of a high field magnet. Stable levitation of one magnet by another with no energy input is usually prohibited by Earnshaw's Theorem. However, the introduction of diamagnetic material at special locations can stabilize such levitation. A magnet can even be stably suspended between (diamagnetic) fingertips.
[S0021-8979(00)68408-6]

INTRODUCTION

Diamagnetic substances include water, protein, diamond, DNA, plastic, wood, and many other common substances usually thought of as nonmagnetic. Bismuth and graphite are the elements with the strongest diamagnetism, about 20 times greater than water. Even for these elements, the magnetic susceptibility χ is exceedingly small, $\chi = -17\times 10^{-5}$.

With powerful magnets, the tiny forces involved are enough to levitate chunks of diamagnetic materials, including blobs of water, plants, and living things such as a frog (Fig. 1).[1,2] The lifting force can cancel gravity throughout the body making this levitation a good approximation of weightlessness for some low gravity experiments.

Situated at the right location, diamagnetic material can stabilize the levitation of permanent magnets. Recently, levitation of a permanent magnet using the diamagnetism of human fingers ($\chi \approx -10^{-5}$) was demonstrated[3] (Fig. 2). This approach can be used to make very stable permanent magnet levitators that work at room temperature without superconductors and without energy input.

EARNSHAW'S THEOREM

At first glance, static magnetic levitation appears to contradict Earnshaw's theorem.[4] Earnshaw discovered something simple and profound. Particles which interact by any type or combination of $1/r^2$ forces can have no stable equilibrium position. Earnshaw's theorem depends on a mathematical property of $1/r$ type energy potentials. The Laplacian of any sum of $1/r$ type potentials is zero, or $\nabla^2 \Sigma k_i/r_i = 0$. This means that at any point where there is force balance ($-\nabla \Sigma k_i/r_i = 0$), the equilibrium is unstable because there can be no local minimum in the potential energy. Instead of a minimum in three dimensions, the energy potential surface is a saddle. If the equilibrium is stable in one plane, it is unstable in the orthogonal direction.

Earnshaw's theorem applies to a test particle, charged and/or a magnet, located at some position in free space with only divergence- and curl-free fields. No combination of electrostatic, magnetostatic, or gravitational forces can create the three-dimensional potential well necessary for stable levitation in free space. The theorem also applies to any rigid array of magnets or charges.

An equivalent way to look at the magnetic case is that the energy of a fixed magnetic dipole $\mathbf{M}$ in a field $\mathbf{B}$ is $U = -\mathbf{M}\cdot\mathbf{B}$ and depends only on the components of $\mathbf{B}$. However, for magnetostatic fields, $\nabla^2 \mathbf{B} = 0$ and the Laplacian of each component is zero in free space and so $\nabla^2 U = 0$ and there is no local energy minimum. There must be some loopholes though, because magnets above superconductors, spinning magnet tops, the frog, and the magnet configuration described in this paper do stably levitate.

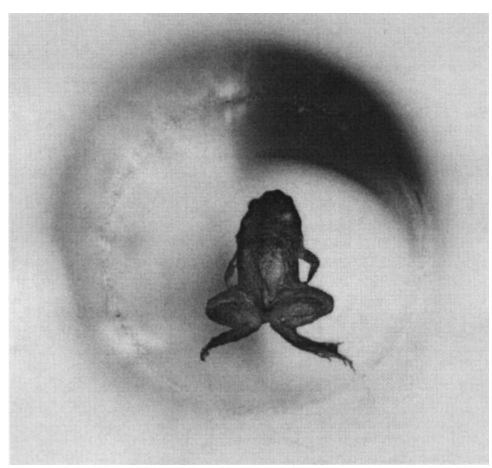


FIG. 1. Frog levitated in stable zone of a 16 T magnet.

[a] Electronic mail: msimon@physics.ucla.edu
[b] Electronic mail: geim@sci.kun.nl

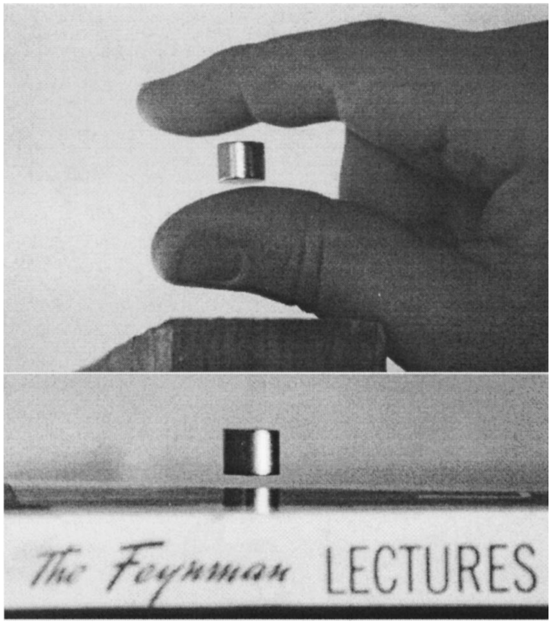


FIG. 2. *Top:* Levitation of a magnet 2.5 m below an unseen 11 T superconducting solenoid stabilized by the diamagnetism of fingers ($\chi \approx 10^{-5}$). *Bottom:* Demonstrating the diamagnetism of our favorite text explaining diamagnetism.

BEYOND EARNSHAW

Earnshaw's theorem does not consider magnetic materials except for hard fixed magnets. Ferro- and paramagnetic substances align with the magnetic field and move toward field maxima. Likewise, dielectrics are attracted to electric field maxima. Since field maxima only occur at the sources of the field, levitation of paramagnets in free space is not possible. (A paramagnet can be made to act like a diamagnet by placing it in a stronger paramagnetic fluid.) Diamagnets are dynamic in the sense that their magnetization changes with the external field. They are repelled by magnetic fields and attracted to field minima. Since local minima can exist in free space, levitation is possible for diamagnets.

Soon after Faraday discovered diamagnetic substances, and only a few years after Earnshaw's theorem, Lord Kelvin showed theoretically that diamagnetic substances could levitate in a magnetic field.[5] In this case the energy depends on $B^2 = \mathbf{B} \cdot \mathbf{B}$ and the Laplacian of B^2 can be positive. In fact $\nabla^2 B^2 \geqslant 0$.[1]

Braunbek[6] exhaustively considered the problem of static levitation in 1939. His analysis allowed for materials with a dielectric constant ϵ and permeability μ different than 1. He showed that stable static levitation is possible only if materials with $\epsilon < 1$ or $\mu < 1$ are involved. Since he believed there are no materials with $\epsilon < 1$, he concluded that stable levitation is only possible with the use of diamagnetic materials.

Braunbek went further than predicting diamagnetic levitation. He figured out the necessary field configuration for stable levitation and built an electromagnet which levitated small pieces of diamagnetic graphite and bismuth.[6]

Superconducting levitation, first achieved in 1947 by Arkadiev,[7] is consistent with Braunbek's theory because a superconductor acts like a perfect diamagnet with $\chi = -1$ and $\mu = 0$.

The only levitation that Braunbek missed is spin-stabilized magnetic levitation of a spinning magnet top over a magnet base which was invented by Roy Harrigan.[8] Braunbek argued that if a system is unstable with respect to translation of the center of mass, it will be even more unstable if rotations are also allowed. This sounds reasonable but we now know that imparting an initial angular momentum to a magnetic top adds constraints which have the effect of stabilizing a system which would otherwise be translationally unstable.[9,10] However, this system is no longer truly static though once set into motion, tops have been levitated for 50 h in high vacuum with no energy inputs.

The angular momentum and precession keep the magnet top aligned antiparallel with the local magnetic field direction making the energy dependent only on the magnitude $|\mathbf{B}| = [\mathbf{B} \cdot \mathbf{B}]^{1/2}$. Repelling spinning dipoles can be levitated near local field minima. Similar physics applies to magnetic gradient traps for neutral particles with a magnetic moment due to quantum spin.[11] The diamagnetically stabilized floating magnets described below stay aligned with the local field direction and also depend only on the field magnitude.

HOW TO FLY A FROG

Diamagnetic materials develop persistent atomic currents which oppose externally applied magnetic fields. The energy of a diamagnetic material with volume V and magnetic susceptibility χ in a magnetic field $\mathbf{B}$ is

$$U = -\frac{\chi B^2 V}{2\mu_0} + mgz. \tag{1}$$

To balance the force of gravity, we require that

$$\frac{\chi V}{2\mu_0} \nabla B^2 = mg\hat{\mathbf{e}}_z \quad \text{or} \quad B\nabla B = \mu_0 g \frac{\rho}{\chi} \hat{\mathbf{e}}_z, \tag{2}$$

where ρ is the mass density of the material to be levitated and $\hat{e}_z$ is the unit vector in the vertical direction. Stability requires that at the levitation point,

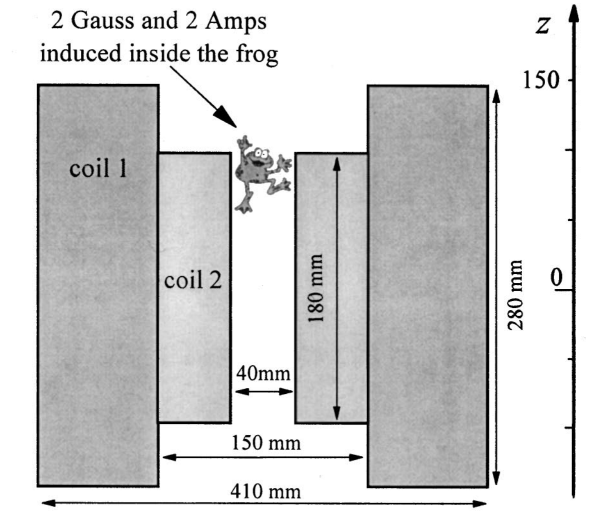
$$\nabla^2 U = -\frac{\chi V}{2\mu_0} \nabla^2 B^2 > 0, \tag{3}$$

and we see that only diamagnets, which have $\chi < 0$, can satisfy the stability condition. This stability condition, while necessary, is not sufficient. For stability, we must have positive curvature in the energy surface in every direction. We can write this condition for diamagnets as

$$\frac{\partial^2 B^2}{\partial z^2} > 0 \quad \text{vertical stability,} \tag{4}$$

$$\frac{\partial^2 B^2}{\partial x^2} > 0, \quad \frac{\partial^2 B^2}{\partial y^2} > 0 \quad \text{horizontal stability.} \tag{5}$$

Taking advantage of the irrotational and divergenceless nature of magnetostatic fields in free space, we can expand the field around the levitation point in terms of the B_z component and its derivatives. For a cylindrically symmetric geometry,



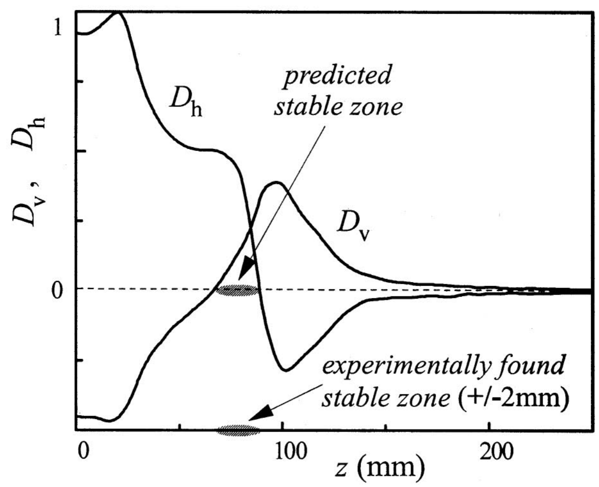


FIG. 3. Vertical and horizontal stability curves for diamagnetic levitation in Bitter magnet. Levitation is stable where both curves are positive and the lifting force matches the weight.

$$B_z = B_0 + B' z + \tfrac{1}{2} B'' z^2 - \tfrac{1}{4} B'' r^2 + \cdots , \tag{6}$$

$$B_r = -\tfrac{1}{2} B' r - \tfrac{1}{2} B'' r z + \cdots , \tag{7}$$

where

$$B' = \frac{\partial B_z}{\partial z}, \text{ and } B'' = \frac{\partial^2 B_z}{\partial z^2}, \tag{8}$$

and the derivatives are evaluated at the levitation point. Then

$$B^2 = B_0^2 + 2 B_0 B' z + \{ B_0 B'' + B'^2 \} z^2 + \tfrac{1}{4} \{ B'^2 - 2 B_0 B'' \} r^2 + \cdots . \tag{9}$$

The vertical and horizontal stability conditions become

$$D_v \equiv B_0 B'' + B'^2 = B_0 B'' + \left[\frac{\rho g \mu_0}{\chi B_0} \right]^2 > 0, \tag{10}$$

$$D_h \equiv B'^2 - 2 B_0 B'' = \left[\frac{\rho g \mu_0}{\chi B_0} \right]^2 - 2 B_0 B'' > 0. \tag{11}$$

Both functions are positive around the inflection point of B_z where $B'' = 0$ and this is the region where diamagnets levitate. The stability functions for a Bitter magnet at the Nijmegen HFML are plotted in Fig. 3 which also shows where the frog levitates.

TABLE I. Values of χ for some diamagnetic materials, $B\nabla B$, and the field required at the center of a 10 cm long solenoid for levitation.

Material	$-\chi(\times 10^{-6})$	$B\nabla B$ T^2	B_c (T)
Water	8.8	1400	11.85
Bismuth metal	170	703	8.4
Graphite rod	160	164	4.1
Pyrolytic graphite $\perp$ axis	450	61	2.5
Pyrolytic graphite $\parallel$ axis	85		

For a thin solenoid of length L and field B_c in the center, $B\nabla B \approx B_c^2/L$ at $L/2$ from the center where the inflection point is. Table I gives some typical values for diamagnetic substances and the field required at the center of a 10 cm long solenoid to levitate them. For a fat solenoid or current loop of radius R, the inflection point is at $z = R/2$.

Most organic materials have a diamagnetic susceptibility near that of water. It is interesting to consider use of this technique as a means to simulate reduced gravity on earth. Because the force acts on every volume element gravity can be nearly canceled throughout the body. Research with plants and small animals animals is now ongoing. Existing technology can handle objects up to 15 cm in diameter.[2] Note that the induced magnetization currents in levitating objects are of the order of a few amps. Nevertheless, they pose no threat and cause no heat because they derive from dissipationless electron orbital motion. Levitation of a person would require a new magnet design with a field of about 40 T and energy consumption of about GW.

MAGNET LEVITATION WITH DIAMAGNETIC STABILIZATION

We know from Earnshaw's Theorem that if we place a magnet in the field of a fixed lifter magnet where the magnetic force balances gravity and it is stable radially, it will be unstable vertically. Boerdijk used graphite below a suspended magnet to stabilize the levitation.[12] Ponizovskii used pyrolytic graphite in a configuration similar to the vertically stabilized levitator described here.[13] As seen in Table I, the best solid diamagnetic material is pyrolytic graphite which forms in layers and has an anisotropic susceptibility (and thermal conductivity). It has much higher susceptibility perpendicular to the sheets than parallel.

It is also possible to levitate a magnet at a location where it is stable vertically but unstable horizontally. In that case a hollow diamagnetic cylinder can be used to stabilize the horizontal motion.[3]

The potential energy U of the floating magnet with dipole moment $\mathbf{M}$ in the field of the lifter magnet is,

$$U = -\mathbf{M} \cdot \mathbf{B} + mgz = -MB + mgz, \tag{12}$$

where mgz is the gravitational energy. The magnet will align with the local field direction because of magnetic torques and therefore the energy is only dependent on the magnitude of the magnetic field, not any field components.

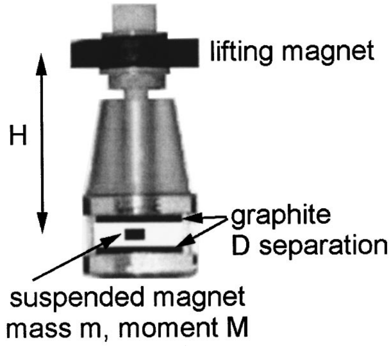


FIG. 4. Diamagnetically stabilized magnet levitation geometry.

Expanding the field magnitude of the lifter magnet around the levitation point using Eqs. (6), (7), and (8) above and adding two new terms $C_z z^2$ and $C_r r^2$ which represent the influence of diamagnets to be added and evaluated next, the potential energy of the floating magnet is

$$U=-M\left[B_0+\left\{B'-\frac{mg}{M}\right\}z+\frac{1}{2}B''z^2 + \frac{1}{4}\left\{\frac{B'^2}{2B_0}-B''\right\}r^2+\cdots\right]+C_z z^2+C_r r^2. \tag{13}$$

At the levitation point, the expression in the first curly braces must go to zero. The magnetic field gradient balances the force of gravity

$$B'=\frac{mg}{M}. \tag{14}$$

The conditions for vertical and horizontal stability are

$$K_v\equiv C_z-\tfrac{1}{2}MB''>0 \quad \text{vertical stability}, \tag{15}$$

$$K_h\equiv C_r+\frac{1}{4}M\left\{B''-\frac{B'^2}{2B_0}\right\}=C_r+\frac{1}{4}M\left\{B''-\frac{m^2g^2}{2M^2B_0}\right\}>0$$

$$\text{horizontal stability}. \tag{16}$$

Without the diamagnets, setting $C_r=0$ and $C_z=0$, we see that if $B''<0$ creating vertical stability, then the magnet is unstable in the horizontal plane. If the curvature is positive and large enough to create horizontal stability, then the magnet is unstable vertically.

When a magnet approaches a weak diamagnetic sheet of relative permeability $\mu=1+\chi\approx1$ we can solve the problem outside the sheet by considering an image current induced in the material but reduced by the factor $(\mu-1)/(\mu+1)\approx\chi/2$. (If the material were instead a perfect diamagnet such as a superconductor with $\chi=-1$ and $\mu=0$, then an equal and opposite image is created as expected.) To take the finite size of the magnet into account we should treat the magnet and image as ribbon currents but here, for simplicity, we will use a dipole approximation which is valid away from the plates.

The energy of the floater dipole M in the field B_i of the induced dipole is

$$U_i=-\tfrac{1}{2}\mathbf{M}\cdot\mathbf{B_i}. \tag{17}$$

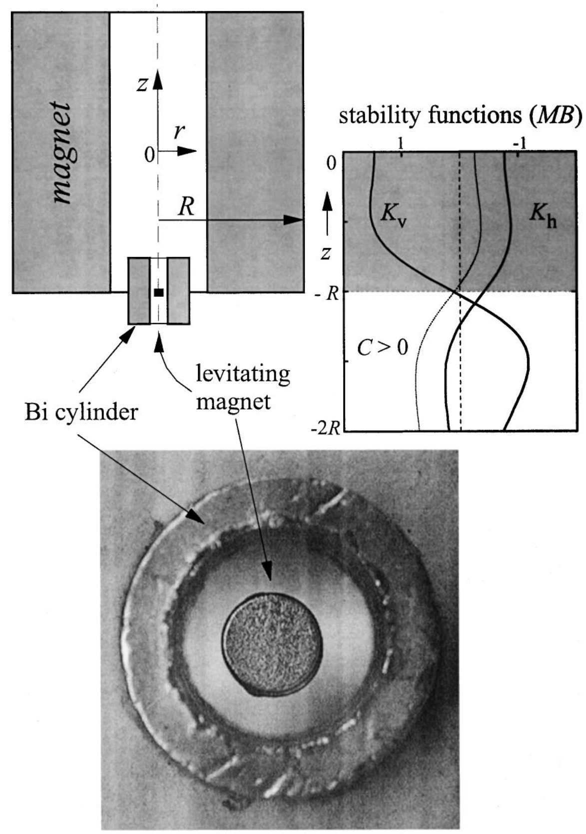


FIG. 5. Vertical and horizontal stability curves for magnet levitation showing the stabilizing effect of a diamagnetic cylinder with an inner diameter of 8 mm and the levitation geometry. Magnet levitation is stable where both curves are positive and the magnetic lifting force matches the weight of the magnet.

Consider first the case where $B''>0$ and is large enough to create horizontal stability. Figure 4 shows the geometry. Adding diamagnetic plates above and below the floating magnet with a separation D and expanding the fields from the induced dipoles around the levitation point in between the two plates gives the energy due to the two diamagnetic plates as

$$U_{\text{dia}}\equiv C_z z^2=\frac{6\mu_0M^2|\chi|}{\pi D^5}z^2. \tag{18}$$

From the stability conditions [Eqs. (15) and (16)], we see that levitation can be stabilized at the point where $B'=mg/M$ if

$$\frac{12\mu_0M|\chi|}{\pi D^5}>B''>\frac{(mg)^2}{2M^2B_0}. \tag{19}$$

This puts a limit on the diamagnetic gap spacing

$$D<\left\{\frac{12\mu_0M|\chi|}{\pi B''}\right\}^{1/5}<\left\{\frac{24\mu_0B_0M^3|\chi|}{\pi(mg)^2}\right\}^{1/5}. \tag{20}$$

If we are far from the lifter magnet field, we can consider it a dipole moment M_L at a distance H from the floater. Then, the condition for stability and gap spacing at the levitation point is

$$D < H \left\{ 2|\chi| \frac{M}{M_L} \right\}^{1/5}. \tag{21}$$

A stronger lifting dipole further away (larger H) allows a larger gap or use of weaker diamagnetic material.

The fingertip stabilized levitation was achieved using a 1 m diam 11 T superconducting solenoid 2.5 m above the levitated magnet where the field was 500 G. Using regular graphite and an inexpensive ceramic lifter magnet it is possible to make a very stable levitator about 5 cm tall with a gap of about 4.4 mm for a 3.175 mm thick 6.35 mm diam NIB magnet. Using pyrolytic graphite, the gap increases to almost 6 mm for the same magnet. This simple design (similar to Fig. 4) could find wide applications.

Finally, consider the case just above the inflection point where $B'' < 0$. A hollow diamagnetic cylinder with inner diameter D as shown in Fig. 5 produces an added energy term

$$U_{\text{dia}} \equiv C_r r^2 = \frac{45\mu_0|\chi|M^2}{16D^5} r^2. \tag{22}$$

Near the inflection point the horizontal stability condition becomes

$$\frac{45\mu_0|\chi|M^2}{2D^5} > \frac{MB'^2}{B_0} = \frac{m^2g^2}{MB_0}, \tag{23}$$

$$D < \left\{ \frac{45\mu_0 B_0 M^3|\chi|}{2(mg)^2} \right\}^{1/5}. \tag{24}$$

This type of levitator can also be implemented on a tabletop using a large diameter ceramic ring magnet as a lifter.

Other configurations for diamagnetically stabilized magnet levitation are possible and rotational symmetry is not required.

ACKNOWLEDGMENTS

We would like to acknowledge useful discussions with Michael Berry and Lee Heflinger.

[1] M. V. Berry and A. K. Geim, Eur. J. Phys. **18**, 307 (1997), and http://www-hfml.sci.kun.nl/hfml/levitate.html.
[2] A. Geim, Phys. Today **51**, 36 (1998).
[3] A. K. Geim, M. D. Simon, M. I. Boamfa, and L. O. Heflinger, Nature (London) **400**, 323 (1999).
[4] S. Earnshaw, Trans. Cambridge Philos. Soc. **7**, 97 (1842).
[5] W. Thomson, *Reprint of Papers on Electrostatics and Magnetism* (Macmillan, London, 1872), Vol. XXXIII, pp. 493–499, and Vol. XXXIV, pp. 514–515.
[6] W. Braunbek, Z. Phys. **112**, 753 (1939); Z. Phys. **112**, 764 (1939).
[7] V. Arkadiev, Nature (London) **160**, 330 (1947).
[8] R. M. Harrigan, U.S. Patent No. 4 382 245, 1983.
[9] M. D. Simon, L. O. Heflinger, and S. L. Ridgway, Am. J. Phys. **65**, 286 (1997).
[10] M. V. Berry, Proc. R. Soc. London, Ser. A **452**, 1207 (1996).
[11] V. V. Vladimirskii, Sov. Phys. JETP **12**, 740 (1961); W. Paul, Rev. Mod. Phys. **62**, 531 (1990).
[12] A. H. Boerdijk, Philips Res. Rep. **11**, 45 (1956); Philips Tech. Rev. **18**, 125 (1956/57).
[13] V. M. Ponizovskii, Prib. Tekh. Eksp. **4**, 7 (1981).